A Sine Intuitionistic Fuzzy Information Measure with their Applications in MADM Problems

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Abstract. A new Sine intuitionistic fuzzy information measure along with its validation is suggested. We have also studied its major properties. Further, a new MADM technique depending upon the weighted correlation coefficients and the suggested intuitionistic fuzzy information measure is proposed along with the necessity of the proposed method. The effectiveness of the method is described by using numerical examples. We have also shown the performance of our proposed method in contrast to some previously present methods in the literature.

Introduction

In our day to day life, we commonly face the problems related with unknown, inaccurate and vague information, like very fast speed, very clever, etc. So, it is very difficult to study such uncertain data with the help of old classical methods. Previously, the probability was the only method to deal with such type of problems. But the problem here is that to calculate the uncertainty with probability method, data must be in crisp form. In order to remove this hurdle or difficulty in the analysis of such data, Zadeh [38] suggested an effective notion of fuzzy set theory. In the early times, many generalizations of the fuzzy sets were proposed by many experimenters. Lately, they reached at the conclusion that the complete information about the inaccurate data(information) cannot be analysed by the fuzzy theory alone. Then, a very famous researcher, Atanassov [2], generalized it into intuitionistic fuzzy theory and this generalization suggested by Atanassov [2] proved to be very effective. The notion of intuitionistic fuzzy sets proposed by Atanassov [2] comprises of two factors, i.e. the membership degree ($\upsilon \in [0, 1]$) and the non-membership degree ($\omega \in [0, 1]$) with the necessary condition $0 \leq (\upsilon + \omega) \leq 1$. A new factor known as `intuitionistic index' or `hesitancy degree' i.e. τ is also included in the previously present structure of the fuzzy sets along with the condition that $\upsilon + \omega + \tau = 1$.

The first generalization in the context of intuitionistic fuzzy sets was proposed by Bhandari and Pal [?]. Later on, Burillo and Bustince [3] proposed an information measure with the help of intuitionistic fuzzy settings. Then, further more generalized measure regarding intuitionistic fuzzy set theory was proposed by many researchers such as Hung and Yang [11], Vlachos and Sergiadis [34] and Zhang and Jiang [39] with help of various operations among intuitionistic fuzzy sets. Therefore, intuitionistic fuzzy sets proved to be more prominent to tackle the inaccurate daily life problems. Intuitionistic fuzzy measure depending on various parameters was suggested by Joshi and Kumar [26]. And a entropy to determine the level of distance among intuitionistic fuzzy sets with help of Renyi-Tsallis information measure was proposed by Rakhi and Kumar [40].

A very important part of our real life is decision making because in our day to day life, we have to take various decisions daily. Based upon the complication of the problem, decision making can be analyticsl or impulsive or a mixture of the two. Thus, it could be defined as the process to find out the best substitute from a set of different substitutes. The process of finding out a best substitute with help of collecting the information comprises of various procedural steps for reaching at our final decision. The procedural steps which are to be followed during the process of decision making are: initially the information or data is collected, selection of various substitutes, then the different selected substitutes are allotted their weights along their attributes and at last, with the help of suitable method the best substitute is chosen. In some of the cases, the involvement of personal feelings may lead to inaccuracy. In the selection of a substitute for a decision process, various factor should be studied or taken into consideration. For example, a business man want to buy a new house in a society, the he will arrive to his decision after analyzing various attributes such as safety, facing of the house, neighbourhood, connectivity to the city and cost of the house. In the process of studying the finite set of substitutes, various decision makers can include themselves by allotting their values to very attribute. Then, the best and most suitable substitute can be calculated by the process of ranking. Therefore, these problems are called multi attribute decision making (MADM) problems.

Many techniques have been proposed by various experimenters for solving multi attribute decision making problems with the help of intuitionistic fuzzy sets. IFSs (Intuitionistic fuzzy sets) are used to represent the properties of substitutes in these method whereas IFN (intuitionistic fuzzy number) denotes the attribute weight and the degree of closeness with ideal solution can be calculated by score function. But the adequate information about substitute might not be provided by these values. In order to overcome this problem, we propose a MADM technique depending upon the weighted correlation coefficients with the help of an under IF settings. The most suitable substitute will be choosen on the basis of ranking of substitutes. The major goals of introducing the paper are :

1. To propose a new entropy in intuitionistic fuzzy sets environment.
2. To suggest a new MADM depending upon weighted correlation coefficients.

In order to get our goal, the paper is presented as follows:

1 Basic Concepts and Definitions

Here, we have provided some of the preliminaries of the fuzzy sets and intuitionistic fuzzy sets.

Definition 2.1. (Refer to [38]) Consider an universal set of discourse $Y = \{h_1, h_2, \ldots, h_k\}$, the fuzzy set $\tilde{L}$ is provided by :

$$\tilde{L} = \left\{ \left\langle h_i, \upsilon_{\tilde{L}}(h_i) \right\rangle \mid h_i \in Y \right\} \tag{2.1}$$

here, $\nu_{\tilde{L}}(h_i)$ denotes the membership degree of hi and is given by $\nu_{\tilde{L}} : Y \to [0,\ 1]$

Later on, Atanassov [2] extended this notion of fuzzy sets into intuitionistic fuzzy sets as given below:

Definition 2.2. (Refer to [2]) Consider an universal set of discourse $Y = \{h_1, h_2, ..., h_k\}$,the intuitionistic fuzzy set L is provided by :

$$\tilde{L} = \{\langle h_i, \upsilon_L(h_i), \omega_L(h_i)\rangle \mid h_i \in Y,\} \quad (2.2)$$

here, $\nu_L(h_i)$ and $\omega_L(h_i)$ represents the membership degree and non-membership degree of h_i respectively and is given by :

$$\upsilon_L : Y \to [0, 1] \text{ and } \omega_L(h_i) : Y \to [0, 1] \quad (2.3)$$

along with the necessary condition that $0 \leq (\upsilon_L + \omega_L) \leq 1, \ \forall h_i \in Y.$

A new factor known as 'intuitionistic index' or 'hesitancy degree' i.e. τ_L is also included in the previously present structure of the fuzzy sets along with the condition that $\upsilon_L + \omega_L + \tau_L = 1$. It is clear that $\omega_L = 1 - \upsilon_L$ if $\tau_L = 0$; $\forall h_i \in Y$, which results in turning an IFS L to a simple fuzzy set. Thus, we can say that fuzzy sets are special case of intuitionistic fuzzy sets.

Further, in the whole paper the set of intuitionistic fuzzy sets on Y will be represented by IFS(Y) whereas the set of fuzzy set on Y will be represented by FS(Y)

Definition 2.3. (Refer to [8]) Consider L, M $\in$ IFS(Y) provided by

$$L = \{\langle h_i, \upsilon_L(h_i), \omega_L(h_i)\rangle\} \mid h_i \in Y\}, \quad (2.4)$$

$$M = \{\langle h_i, \upsilon_M(h_i), \omega_M(h_i)\rangle \mid h_i \in Y\};$$

given below are the operations and relations that follows:

1. $L \subseteq M$ iff $\upsilon_L \leq \upsilon_M$, $\omega_L \geq \omega_M$ for $\upsilon_M < \leq \omega_K$ **OR** $\upsilon_L \geq \upsilon_M$, ω_L for $\upsilon_M < \geq \omega_K$
2. L = M iff $L \subseteq M$ and $M \subseteq L$.
3. $L^c = \{\langle hi, \omega_L(h_i)\nu_L(h_i)\rangle \mid h_i \in Y\}$, where L^c represents the complement of the set L.
4. $L \cap M = \{\langle \nu_L(h_i) \cap \nu_M(h_i) \text{ and } \omega_L(h_i) \cup \omega_M(h_i)\rangle \mid h_i \in Y\};$
5. $L \cup M = \{\langle \nu_L(h_i) \cup \nu_M(h_i) \text{ and } \omega_L(h_i) \cap \omega_M(h_i)\rangle \mid h_i \in Y\}.$

Definition 2.4. (Refer to [11]) A function F : IFS(Y) $\to$ [0; 1], is said to be an entropy on IFS(Y) it is follows the below mentioned conditions:

1. **Sharpness:** F(L) = 0 iff L is a crisp set, that is, $\upsilon_L(h_i) = 0$, $\omega_L(h_i) = 1$; or $\upsilon_L(h_i) = 1$; $\omega_L(h_i) = 0 \forall h_i \in Y$;
2. **Maximality:** If $\upsilon_L(h_i) = \omega_L(h_i) = \tau_L(h_i) = \frac{1}{3}$, $\forall h_i \in Y$, then F(L) gains its maximum value.
3. **Symmetry** $F(L) = F(L^c)$, here complement of L is represented by L^c;
4. **Resolution:** $F(L) \leq F(M)$ if L is sharper than M, i.e., $\upsilon_L \leq \upsilon_M$ and $\omega_L \leq \omega_M$ for maximum $(\upsilon_M, \omega_M) \leq \frac{1}{3}$ and $\upsilon_L \geq \upsilon_M$ and $\omega_L \geq \omega_M$ for minimum $(\upsilon_M, \omega_M) \geq \frac{1}{3}$.

Definition 2.5. (Refer to [10]) The correlation coefficient for some L, M $\in$ IFS(Y) is provided by:

$$V(L, M) = \frac{D(L, M)}{\sqrt{(S(L).S(M))}} \quad (2.5)$$

here, correlation of the 2 IFSs is given by $D(L, M) = \sum_{i=1}^{k} (\nu_L(h_i)\upsilon_M(h_i) + \omega_L(hi)\omega_M(h_i))$ and the informational intuitionistic energies are provided by $S(L) = \sum_{i=1}^{k} (\upsilon_L(h_i)^2 \omega_L(h_i)^2 + \omega_L(hi)^2 \omega_{:L}(h_i)^2$ and
$S(M) = \sum_{i=1}^{k} (\nu_M(h_i)^2 \omega_M(h_i)^2$ respectively.

The following conditions are satisfied by the correlation coefficient:

1. $0 \le V(L, M) \le 1$.
2. $V(L, M) = V(M, L)$.
3. $V(L, M) = 1$ if $L = M$

Considering these notions and suggestions in mind, now we will introduce a new sine intuitionistic fuzzy information measure in the succeeding section.

2 A New Intuitionistic Fuzzy Information Measure

3.1 Definition

For any $L \in IFS(Y)$, sine intuitionistic fuzzy information measure is defined as

$$H_{sine}(L) = \frac{1}{3k}\sum_{i=1}^{k} \sin\pi(\omega_L(h_i)) + \sin\pi(\tau_L(h_i)) \quad (3.1)$$

where $0 \le h_i \le 1$.

3.2 Justification

In this subsection, we will justify an essential property for the existence of suggested measure.

Property 3.1 According to the statement I_3, we have

$$\left|\upsilon_L(h_i) - \frac{1}{3}\right| + \left|\omega_L(h_i) - \frac{1}{3}\right| + \left|\tau_L(h_i) - \frac{1}{3}\right| \ge \left|\upsilon_M(h_i) - \frac{1}{3}\right| + \left|\omega_L(h_i) - \frac{1}{3}\right| + \left|\tau_L(h_i) - \frac{1}{3}\right| \quad (3.2)$$

and

$$\left(\upsilon_L(h_{i)} - \frac{1}{3}\right)^2 + \left(\omega_L(h_{i)} - \frac{1}{3}\right)^2 + \left(\tau_L(h_{i)} - \frac{1}{3}\right)^2 \ge \quad (3.3)$$
$$\left(\upsilon_M(h_{i)} - \frac{1}{3}\right)^2 + \left(\omega_M(h_{i)} - \frac{1}{3}\right)^2 + \left(\tau_M(h_{i)} - \frac{1}{3}\right)^2$$

Proof. If $\upsilon_L(h_i \le \upsilon_M(h_i$ and $\omega_L h_i) \le \omega_M(h_i)$ with maximum $\{\upsilon_M(h_i), \omega_M(h_i\} \le \frac{1}{3}$, then $\nu_L(h_i \le \nu_m(h_i \le \frac{1}{3}, \omega_L(hi) \le \frac{1}{3}$ and $\tau_{:L}(h_i) \ge \tau_M(h_i) \ge \frac{1}{3}$ therefore (3.2) and (3.3) holds.

In the same way we can show that, if $\upsilon_L(h_i \ge \upsilon_M(h_i$ and $\omega_L(h_i) \ge \omega_M(h_i)$ maximum $\{\upsilon_M(h_i, \omega_M(h_i\} \ge \frac{1}{3}$, the equations (3:2) and (3:3) are satisfied.

Theorem 3.2 Prove that the measure (3:1) is valid.

Proof. For establishing (3:1) as a valid measure, we show that it follows the conditions in Definition (2.4)

1. Sharpness: If $H_{sine}(L) = 0$ then,

$$\sin\pi(\upsilon_L(h_i)) + \sin\pi(\omega_L(h_i)) + \sin\pi(\tau_L(h_i)) = 0 \quad (3.4)$$

As $0 \le h_i \le 1$, which is possible only in the below mentioned cases:

(a) Either $\upsilon_L(h_i) = 1$, that is, $\omega_L(h_i) = \tau_L(h_i) = 0$ or

(b) $\omega_L(h_i) = 1$, that is, $\upsilon_L(h_i) = \tau_L(h_i) = 0$ or

(c) $\tau_L(h_i) = 1$, that is, $\upsilon_L(h_i) = \omega_L(h_i) = 0$.

In each case mentioned above, $H_{sine}(L) = 0$ which confirms that L is a crisp set.

Conversely, let L be a crisp set then either $\upsilon_L(h_i) = 1$ and $\omega_L(h_i) = \tau_L(h_i) = 0$ or $\omega_L(hi) = 1$ and $\upsilon_L(h_i) = \tau_L(h_i) = 0$ or $\tau_L(h_i) = 1$ and $\upsilon_L(h_i) = \omega_L(h_i) = 0$.

Therefore

$$\sin \pi(\upsilon_L(h_i)) + \sin \pi(\omega_L(h_i)) + \sin \pi(\tau_L(h_i)) = 0 \qquad (3.5)$$

$\Rightarrow H_{sine}(L) = 0$ iff L is a crisp set.

2. Maximality: As we know that $\upsilon + \omega + \tau = 1$, in order to attain the maximum value of IF entropy $H_{sine}(L)$, we write $\upsilon(\upsilon_L, \omega_L, \tau_L) = \upsilon_L(h_i) + \omega_L(hi) + \tau_L(h_i) - 1$ and considering the Lagranges' multiplier λ,

Let,

$$\psi(\upsilon_L, \omega_L, \tau_L) = H_{sine}(\upsilon_L, \omega_L, \tau_L) + \lambda\, \varphi(\upsilon_L, \omega_L, \tau_L) \qquad (3.6)$$

For calculating the maximum value of $H_{sine}(L)$, (3:6) to be differentiated partially w.r.t. υ_L, ω_L, τ_L, $\lambda$and then equated to zero, gives $\upsilon_L(h_i) = \omega_L(h_i) = \tau_L(h_i) = \frac{1}{3}$. It is observed that all the first order partial derivatives becomes zero iff $\upsilon_L(h_i) = \omega_L(h_i) = \tau_L(h_i) = \frac{1}{3}$. Now, we will prove that $H_{sine}(L)$ is concave function. In order to do so, its hessian at the stationary point is evaluated, which is given by

$$\hat{H}_{sine}(L) = \frac{\pi^2}{3}\sin\frac{\pi}{3}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \qquad (3.7)$$

$\hat{H}_{sine}(L)$ is a negative definite matrix, which implies that $H_{sine}(L)$ is a concave function.

Therefore, If $\upsilon_L(h_i) = \omega_L(h_i) = \tau_L(h_i) = \frac{1}{3}$, $\forall h_i \in Y$, then F(L) gains its maximum value.

3. Symmetry: It is obvious and follows directly from the definition that $H_{sine}(L) = H_{sine}(L^c)$.

4. Resolution: As we know that $H_{sine}(L)$ is a concave function, if maximum $\{\upsilon_M(h_i, \omega_M(h_i\} \le \frac{1}{3}$, so $\upsilon_L(h_i \le \upsilon_M(h_i$ and $\omega_L(h_i) \le \omega_M(h_i)$ which shows $\tau_L(h_i) \ge \tau_M(h_i) \ge \frac{1}{3}$. Hence, from property (3.1), we can say that condition I_3 is satisfied.

In the same way, if minimum $\{\upsilon_M(h_i, \omega_M(h_i\} \ge \frac{1}{3}$, so $\upsilon_L(h_i \ge \upsilon_M(h_i$ and $\omega_L(h_i) \ge \omega_M(h_i)$.

Hence, from property (3.1), we can say that condition I_3 is satisfied.

Hence, it is proved that measure (3.1) is valid intuitionistic fuzzy information measure.

3 Properties

In this section, we will study various properties of our suggested measure.

Theorem. Consider L and M be 2 intuitionistic fuzzy sets on a universal set of discourse Y = $\{h_1, h_2, \ldots, h_k\}$ where $L = \{\langle h_i, \upsilon_L(h_i), \omega_L(h_i) / h_i \in Y\rangle\}$ and $M = \{\langle h_i, \upsilon_M(h_i), \omega_M(h_i) h_i \in Y\rangle\}$ s.t., $\forall hi \in Y$ either $L \subseteq M$ or $K \supseteq L$; then,

$$H_{sine}(L \cup M) + H_{sine}(L \cap M) = H_{sine}(L) + H_{sine}(M). \quad (4.1)$$

Proof. Let us divide Y in two parts i.e., Y_1 and Y_2, s.t.,

$$Y_1 = \{h_i \in Y : L \subseteq M\},\ Y_2 = \{h_i \in Y : L \supseteq M\}. \quad (4.2)$$

$\Rightarrow \forall h_i \in Y_1$,

$$\upsilon_L(h_i) \le \upsilon_M(h_i),\ \omega_L(h_i) \ge \omega_M(h_i) \quad (4.3)$$

$\Rightarrow \forall h_i \in Y_2$,

$$\upsilon_L(h_i) \ge \upsilon_M(h_i),\ \omega_L(h_i) \le \omega_M(h_i) \quad (4.4)$$

From (3.1), we get

$$H_{sine}(L \cup M) = \frac{1}{3k}\sum_{i=1}^{k} \sin\pi(\upsilon_{L\cup M}(h_i)) + \sin\pi(\omega_{L\cup M}(h_i) + \sin\pi(\tau_{L\cup ML}(h_i)) \quad (4.5)$$

$$H_{sine}(L \cup M) = \frac{1}{3k}\left[\begin{array}{l}\sum_{Y_1} \sin\pi(\upsilon_M(h_i)) + \sin\pi(\omega_M(h_i) + \sin\pi(\tau_M\ (h_i)) + \\ \sum_{Y_2} \sin\pi(\upsilon_L(h_i)) + \sin\pi(\omega_L(h_i) + \sin\pi(\tau_L\ (h_i))\end{array}\right] \quad (4.6)$$

In the same way, we can show that

$$H_{sine}(L \cap M) = \frac{1}{3k}\left[\begin{array}{l}\sum_{Y_1} \sin\pi(\upsilon_L(h_i)) + \sin\pi(\omega_L(h_i) + \sin\pi(\tau_L(h_i)) \\ +\sum_{Y_2} \sin\pi(\upsilon_M(h_i)) + \sin\pi(\omega_M(h_i) + \sin\pi(\tau_M(h_i))\end{array}\right] \quad (4.7)$$

By using (4.6) and (4.7), we get

$$H_{sine}(L \cup M) + H_{sine}(L \cap M) = H_{sine}(L) + H_{sine}(M). \quad (4.8)$$

Hence, the theorem is proved.

4 The New Madm Method using Suggested if Measure

Let us suppose a MADM problem with k-non-inferior substitutes provided by W = $(\psi_1, \psi_2, \ldots, \psi_k)$ and a collection of r-attributes provided by $\Gamma = (\Gamma_1, \Gamma_2, \ldots, \Gamma_r)$. We have to select the most suitable ψ_1 (i = 1, 2, …, k) following Γ_j(j = 1, 2, …, r). Intuitionistic fuzzy numbers (IFNs) represents the level at which a specific substitute follows the particular attribute as granted by various decision-makers and are provided by $\tilde{g}_{ij} = (\upsilon_{ij}, \omega_{ij})$, here υ_{ij} and ω_{ij} represents the membership and non-membership degrees which follows the conditions: $0 \le \upsilon_{ij} \le 1$, $0 \le \omega_{ij} \le 1$ and $0 \le \upsilon_{ij} + \omega_{ij} \le 1$. For calculating the value of υ_{ij} and ω_{ij}, we use the technique given by Liu and Wang [28] as:

$$\upsilon_{ij} = \frac{r_{true}(ij)}{S} \text{ and } \omega_{ij} = \frac{r_{false}(ij)}{S} \quad (5.1)$$

here, the number of decision-makers in favour of the ψ_1(i = 1, 2, …, k) are represented by $r_{true}(ij)$ regarding attribute γ_j(j = 1, 2, …,r) whereas the number of decision-makers which opposed the substitute ψ_1(i = 1, 2, …, k) are represented by $r_{false}(ij)$ regarding attribute γ_j(j = 1, 2, …,r) and total number of decision-makers are given by S. The complete problem can be formulated in a IF decision matrix $Y = (\tilde{g}_{ij})_{m\times n}$ written as:

$$Y = (\tilde{g}_{ij})_{m\times n} = \begin{matrix} & x_1 & x_1 & & x_1 \\ M_1 & (\upsilon_{11},\ \omega_{11}) & (\upsilon_{12},\ \omega_{12}) & \ldots & (\upsilon_{1r},\ \omega_{1r}) \\ M_1 & (\upsilon_{11},\ \omega_{11}) & (\upsilon_{12},\ \omega_{12}) & \ldots & (\upsilon_{1r},\ \omega_{1r}) \\ M_1 & \vdots & \vdots & \ldots & \vdots \\ M_1 & (\upsilon_{k1},\ \omega_{k1}) & (\upsilon_{k2},\ \omega_{k2}) & \ldots & (\upsilon_{kr},\ \omega_{kr}) \end{matrix} \qquad (5.2)$$

In solving a MADM problem, the major character is played by the attributes weights. If the attributes weights are allotted in a genuine manner then the most suitable substitute is selected genuinely, but if failed to do so then it may case false choice of suitable substitute. Thus, we can say that in the process of allotting the attributes weights complete justification should be given by the decision makers. Because of the pressure of time, lesser knowledge regarding the domain of problem or due to the complexity, decision makers uses form intervals for expressing them in the place of pinpoint numbers. In order to resolve this issue, the operation of finding the attributes weights is bifurcated into two parts as given below:

5.1 If Attributes Weights are Completely Unknown

If the we do not know about the attribute weights, then the technique suggested by Chen et al. [5] and Ye [37] is used for calculating the attributes as given below:

$$\upsilon_j = \frac{1 - F_j}{r - \sum_{j=1}^{r} F_j},\ j = 1,\ 2,\ \ldots, r, \qquad (5.3)$$

Here $F_j = \frac{1}{r}\sum_{i=1}^{k} H_{sine}(\tilde{g}_{ij})$ and

$$H_{sine}(\tilde{g}_{ij}) = \frac{1}{3k}\sum_{i=1}^{k} \sin\pi(\upsilon_L(h_i)) + \sin\pi(\omega_L(h_i)) + \sin\pi(\tau_L(h_i))$$

5.2. If Attributes Weights Are Partially Known

Commonly, there are many limitations for the attribute weight vector $\mu = (\mu_1, \mu_2, \ldots, \mu_r)$. As stated before, the result provided by the decision makers are not always in the form of pinpoint numbers. So, in this type of situations the information available to us is partial.

Consider the information regarding the attribute weights that we have be represented by V. For the calculation of attribute weights in these type of cases, the minimum entropy principle suggested by Wang and Wang [36] is utilized and is given by:

$$F(\psi_1) = \sum_{j=1}^{r} H_{sine}(\tilde{g}_{ij}) = \sum_{j=1}^{r} \frac{1}{3k}\sum_{i=1}^{k} \sin\pi(\upsilon_L(h_i)) + \sin\pi(\omega_L(h_i)) + \sin\pi(\tau_L(h_i)) \quad (5.4)$$

The weight coefficients regarding the identical attributes must be alike, this is because every attribute is a healthy opposition. Hence, in order to get the prominent attribute weights, the programming model given below is formulated:

$$\min F = \sum_{i=1}^{k} \mu_j \left(\sum_{j=1}^{r} H_{sine}(\tilde{g}_{ij})\right) = \frac{1}{3k}\sum_{i=1}^{k}\sum_{j=1}^{r} \mu_j (\sin\pi(\upsilon_L(h_i))$$
$$+ \sin\pi(\omega_L(h_i)) + \sin\pi(\tau_L(h_i))) \qquad (5.5)$$

The prominent solution is obtained as arg minimum $F = (\mu_1, \mu_2, \ldots, \mu_r)^S$.

5.3. The Suggested MADM Method

The procedural steps of the suggested technique is given below:

1. Find the attributes weights with the help of (5.3) and (5.5).
2. Calculate the ψ^+ and ψ^-as given below:

$$\psi^+ = ((\varphi_1^+, \varsigma_1^+),\ (\varphi_2^+, \varsigma_2^+),\ ...,(\varphi_r^+, \varsigma_r^+)), \quad (5.6)$$

here $(\varphi_j^+, \varsigma_j^+) = (\sup(\upsilon_j(h_i)), \text{in f}(\omega_j(h_i)) = (1,\ 0),\ j = 1,\ 2,\ ...,\ \text{and } h_i \ \in Y.$

and

$$\psi^- = ((\varphi_1^-, \varsigma_1^-),\ (\varphi_2^-, \varsigma_2^-),\ ...,(\varphi_r^-, \varsigma_r^-)), \quad (5.7)$$

here $(\varphi_1^-, \varsigma_1^-) = (\text{in f }\ (\upsilon_j(h_i)),\ \sup(\omega_i(h_i))) = (0,\ 1)\ j\ =\ 1,\ 2,\ ...,\ r \text{ and } h_i \in Y.$

3. With the help of correlation coefficients among IFSs proposed by Gerstenkorn and Manko [10], the correlation coefficients among the substitutes ψ_i(i = 1, 2, …, k) and the preeminent solution ψ^+ is calculated by

$$DN_i(\psi^+, \psi_i) = \frac{D(\psi^+, \psi_i)}{\sqrt{S((\psi^+)S(\psi_i)}} = \frac{\sum_{j=1}^{k} \rho_j \upsilon_{\psi_i}(f_i)}{\sqrt{\sum_{j=1}^{k} \rho_j (\upsilon_{\psi_i}(f_i)^2 + \omega_{\psi_i}(f_i)^2)}} \quad (5.8)$$

And in the same manner, the correlation coefficients among the substitutes ψ_1(i =1, 2, …, k) and the worst solution ψ^- is calculated by

$$DN_i(\psi^+, \psi_i) = \frac{D(\psi^+, \psi_i)}{\sqrt{S((\psi^+)S(\psi_i)}} = \frac{\sum_{j=1}^{k} \rho_j \upsilon_{\psi_i}(f_i)}{\sqrt{\sum_{j=1}^{k} \rho_j (\upsilon_{\psi_i}(f_i)^2 + \omega_{\psi_i}(f_i)^2)}} \quad (5.9)$$

4. Calculate the relative closeness coefficients as given below:

$$T = \frac{DN_i(\psi^-, \psi_i)}{DN_i(\psi^-, \psi_i) + DN_i(\psi^+, \psi_i)} \quad (5.10)$$

5. The substitutes regarding the values of $T_i^{'}s$ are arranged in defending order by the process of ranking. The substitute with the highest value will be considered as most suitable substitute.

6. Examples

In this section, we will provide the applications of MADM technique by using the examples given below:

Case 1. If Attributes weights are unknown.

Example 6.1. Let us take an example of a soft drink manufacturing industry which want to arrange the capable distributors for the supply of crude products which are essential for the manufacturing of soft drink. From the various summoned quotations, we have nominated the 4 quotations which are ψ1, ψ2, ψ3 and ψ4 that are to be ranked. The norms on the basis of which the distributors are to be ranked has been marked by company such as (Γ_1) nature, ((Γ_2) delivery time and ((Γ_3) healthy or unhealthy. For assuring that the selection of distributors should be equitable, a team consisting of DMs having non-identical framework, proficiency, information has been formulated.

The method suggested by Liu and Wang [28] may be used for calculating the values of favourable degrees υ_{ij} and non-favourable degrees ω_{ij} corresponding to the substitutes $\psi_i^{'}$s(i =

1, 2, …, k) following the attributes $\Gamma_j^{'}$ s (j = 1, 2, …, r) simultaneously [Let T = 100]. Consider that decision-makers responds in the form of 'true' or 'false' and their responses are presented in Table 1.

With the help of (5.1), the intuitionistic fuzzy decision matrix regarding to Table 1 is shown in Table 2.

Further, we formulate the IF information matrix regarding Table 2 with the help of (3.1) and the results are shown in Table 3.

The particular calculations are given below:

Table 1: Decision makers responses

	Γ_1 $< r_{true}(i1); r_{false}(i1) >$	**Γ_2 $< r_{true}(i2); r_{false}(i2) >$**	**Γ_3 $< r_{true}(i3); r_{false}(i3) >$**
ψ1	< 40, 30 >	< 55, 35 >	< 25, 60 >
ψ2	< 50, 45 >	< 55, 30 >	< 75, 20 >
ψ3	< 50, 35 >	< 50, 40 >	< 65, 30 >
ψ4	< 85, 10 >	< 60, 25 >	< 70, 25 >

Table 2: IF Decision Matrix

Substitutes	**Evaluation Attributes**		
ψ1	Γ_1	Γ_2	Γ_3
ψ2	< 0.40, 0.30 >	< 0.55, 0.35 >	< 0.60, 0.25 >
ψ3	< 0.50, 0.45 >	< 0.55, 0.30 >	< 0.75, 0.20 >
ψ4	< 0.50, 0.35 >	< 0.50, 0.40 >	< 0.65, 0.30 >
ψ1	< 0.85, 0.10 >	< 0.60, 0.25 >	< 0.70, 0.25 >

Table 3: IF Decision Matrix

Substitutes	**Evaluation Attributes**		
	Γ_1	Γ_2	Γ_3
ψ1	0.8563	0.7292	0.7040
ψ2	0.9958	0.7523	0.7649
ψ3	0.7816	0.7533	0.9000
ψ4	0.5876	0.7040	0.8387
F_j	0.8053	0:7347	0.8019

1. From (5.3), the calculated attribute weight vector is :

$\mu = (\mu_1, \mu_2, \mu_3)^S = (0.2958, 0.4031, 0.3010)^S$.

2. The ψ^+ and ψ^- are provided by:

$$\psi^+ = ((\varphi_1^+, \varsigma_1^+),\ (\varphi_2^+, \varsigma_2^+),\ (\varphi_3^+, \varsigma_r^+)) = ((1,0),(1,0),(1,0));$$

$$\psi^- = ((\varphi_1^-, \varsigma_1^-),\ (\varphi_2^-, \varsigma_2^-),\ (\varphi_3^-, \varsigma_r^-)) = ((1,0),(1,0),(1,0));$$

3. The calculated values of correlation coefficients with the help of (5.8) are

$DN_1(\psi^+, \zeta_1) = 0.8542$, $DN_2(\psi^+, \zeta_2) = 0.8638$,
$DN_3(\psi^+, \zeta_3) = 0.8371$, $DN_4(\psi^+, \zeta_4) = 0.9501$;
$DN_1(\psi^-, \zeta_1) = 0.4960$, $DN_2(\psi^+, \zeta_2) = 0.4558$,
$DN_3(\psi^-, \zeta_3) = 0.5327$, $DN_4(\psi^+, \zeta_4) = 0.2651$.

4. Calculated values of relative closeness coefficients with help of (5.10) are provided by

$$T_1 = 0.3672; T_2 = 0.3454; T_3 = 0.3888; T_4 = 0.2181. \quad (6.1)$$

Then, the substitutes are placed in descending order and we have the ranking order as : $\psi_3 \succ \psi_1 \succ \psi_2 \succ \psi_4$. Hence, ψ_3 is best suitable substitute.

Why The Suggested Method Was Needed?

Here, we will validate the need of the suggested technique. In order to so, the initial step is to define the traditional MADM technique. The steps to be followed are:

1. First two steps of the traditional technique and our suggested technique are alike.
2. Then in the next step, i.e., step 3, the distance of $\psi_i(i = 1; 2, \ldots, k)$ from ψ^+ and ψ^- is computed with the help of various distance measures. Consider $C_i(\psi^+, \psi_i)$ and $C_i(\psi^-, \psi_i)$ represents these distances.
3. The relative closeness coefficients let us say T_i is calculated as given below:

$$T_i = \frac{C_i(\psi^-, \psi_i)}{C_i(\psi^+, \psi_i) + C_i(\psi^-, \psi_i)} \quad (6.2)$$

4. At last, in the final step the substitutes are arranged in the descending order.

Our suggested technique differs from the traditional MADM technique only in the steps 3. The weighted distance measure is used is the traditional MADM technique for calculation of distance among the ideal solutions and substitutes, but in our suggested technique, we have used the weighted correlation coefficients.

Now, various distance measures are used for calculating the above example and results are compared.

1. We begin with using the weighted Hamming distance measure provided by:

$$GC(L,M) = \frac{1}{2}\sum_{j=1}^{r}\left[\rho_j(|\upsilon_L(h_i) - \rho_M(h_i)| + |\omega_L(h_i) - \omega_M(h_i)| + |\tau_L(h_i) - \tau_M(h_i)|)\right]; \quad (6.3)$$

for calculating's the distances among $\psi_i^{'}$s from ψ^+ and ψ^- and the results are given as

$C_1(\psi^+,\zeta_1 = 0.4833$, $C_2(\psi^+, \zeta_2 = 0.3916$, $C_3(\psi^+,\zeta_3= 0.4500$, $4(\psi^+,\zeta_4= 0.2833$; (6.4)

$C_1(\psi^-,\zeta_1 = 0.7000$, $C_2(\psi^-,\zeta_2 = 0.6833$, $C_3(\psi^-,\zeta_3= 0.6500$; $C_4(\psi^+,\zeta_4= .8000$. (6.5)

The computed values of relative closeness coefficients $T_i(i = 1, 2, 3, 4)$ the help of (6.2) are provided by

$$T_1 = 0.5915; T_2 = 0.6356; T_3 = 0.5909; T_4 = 0.7384; \quad (6.6)$$

Then, the subsitutes are placed in descending order and we have the ranking order as :

$\psi_4 \succ \psi_2 \succ \psi_1 \succ \psi_3$. Hence, ψ_4 is best suitable substitute.

2. Further, we solve it by using the technique suggested by Wang and Xin [36]

$$ZY(L, M) = \sum_{j=1}^{r}\rho_j\left(\frac{|\upsilon_L(h_i) - \upsilon_M(h_i)| + |\omega_L(h_i) - \omega_M(h_i)|}{4} + \frac{\max(|\upsilon_L(h_i) - \upsilon_M(h_i)|,\ |\omega_L(h_i) - \omega_M(h_i)|)}{2}\right) \quad (6.7)$$

The calculated distance measures are

$C_1(\psi^+,\zeta_1 = 0.4292$, $C_2(\psi^+, \zeta_2 = 0.3500$, $C_3(\psi^+,\zeta_3 = 0.4042$, $(\psi^+,\zeta_4 = 0.2209$; (6.8)

$C_1(\psi^-,\zeta_1 = 0.6460$ $C_2(\psi^-,\zeta_2 = 0.6417$, $C_3(\psi^-,\zeta_3 = 0.6042$; $C_4(\psi^+,\zeta_4 = 0.7376$. (6.9)

The computed values of relative closeness coefficients $T_i(i = 1, 2, 3, 4)$ the help of (6.2) are provided by

$T_1 = 0.6008$; $T_2 = 0.6470$; $T_3 = 0.5991$; $T_4 = 0.7695$. (6.10)

Then, the subsitutes are placed in descending order and we have the rankning order as : $\psi_4 \succ \psi_2 \succ \psi_1 \succ \psi_3$. Hence, ψ_4 is best suitable substitute.

3. On calculated the example 6.1 with the help of technique suggested by Ye [37], the output we get is

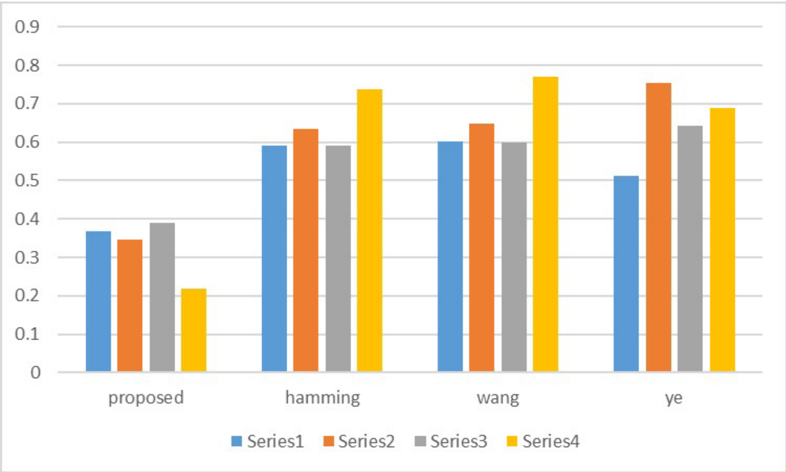
$$\psi_4 \succ \psi_2 \succ \psi_1 \succ \psi_3 \quad (6.11)$$

Whereas, Only the correlation of substitutes along +ve ideal solution is taken into considera-tion

in the Ye's [37] technique.

A Graphical Analysis: In order to get more deep knowledge about above mentioned output, they are plotted as follows:

It is observed that the outcomes of traditional techniques differs along the measures we use. Hence, we have to choose a technique in which the outcomes are independent of the distance measure and are constant. Our suggested method fulfill all this requirements which validates its necessity. (Figure 1.)



Case 2. If Attribute Weights are partially known to us

Example 6.2. Let us take an example of 100 elected representatives of the assembly who want to choose their leader of the house. Let there are 3 substitutes say ψ_1, ψ_2 and ψ_3. Evaluation attributes are (Γ_1) Honest, (Γ_2) Helpful and (Γ_3) Intelligencey.

The IF decision matrix given by DM's is shown in Table 4.

The information matrix regarding to IF decision matrix is provided in Table 5.

Given below is the set of available weight information :

$V = \{0.30 \le \omega_1 \le 0.70; 0.45 \le \omega_2 \le 0.50, 0.25 \le \omega_1 \le 0.40\}$

Table 4: IF Decision Matrix

Substitutes	**Evaluation Attributes**		
ψ1	< 0.70; 0.15 >	< 0.65; 0.30 >	< 0.85; 0.10 >
ψ2	< 0.70; 0.25 >	< 0.55; 0.15 >	< 0.65; 0.25 >
ψ3	< 0.50; 0.40 >	< 0.80; 0.15 >	< 0.45; 0.35 >

Table 5: IF Decision Matrix

Substitutes	**Evaluation Attributes**		
	Γ_1	Γ_2	Γ_3
ψ1	0.5723	0.9000	0.5876
ψ2	0.8387	0.7502	0.6357
ψ3	0.7533	0.6805	0.8221
F_j	0.7214	0.7769	0.6818

The calculated are steps are given below:

1. With the help of (5.5), we formulate a programming model as given below:

$$\min E = 0.7214\omega_1 + 0.7769\omega_2 + 0.6818\omega_3 \qquad (6.12)$$

$$\begin{cases} 0.30 \le \omega_1 \le 0.70 \\ 0.45 \le \omega_2 \le 0.50 \\ 0.25 \le \omega_1 \le 0.40 \\ \omega_1 + \omega_2 + \omega_3 = 1. \end{cases} \qquad (6.13)$$

By using MATLAB, we can get the value of weight vector as:

$\mu = (0.34, 0.27, 0.39)^S$ (6.14)

2. The ψ^+ and ψ^- are provided by:

$$\psi^+ = ((\varphi_1^+, \zeta_1^+), (\varphi_2^+, \zeta_2^+), (\varphi_3^+, \zeta_3^+)) = ((1,0),(1,0),(1,0));$$
$$\psi^- = ((\varphi_1^-, \zeta_1^-), (\varphi_2^-, \zeta_2^-), (\varphi_3^-, \zeta_3^-)) = ((1,0),(1,0),(1,0)).$$

3. The calculated values of correlation coe_cients with the help of (5.8) are

$$C_1(\psi^+, \zeta_1) = 0.7166, C_2(\psi^+, \zeta_2) = 0.9398,$$
$$C_3(\psi^+, \zeta_3) = 0.8547;$$
$$C_1(\psi^-, \zeta_1) = 0.1791, C_2(\psi^-, \zeta_2) = 0.3215,$$
$$C_3(\psi-, \zeta_3) = 0.4395.$$

4. Calculated values of relative closeness coefficients with help of (5.10) are provided by

$$T_1 = 0.1999; T_2 = 0.2548; T_3 = 0.3395: \qquad (6.15)$$

The ranked order is: $\psi_3 \succ \psi_2 \succ \psi_1$ which implies ψ_3 is most suitable substitute.

On calculating this e.g. with help of technique suggested by Chen [4] and Li [27], we attain ψ_1 is most suitable substitute. The results differs because of the different divergence measures taken for computation.

5 Conclusion

A very crucial part is played by IFSs in the solution of MADM problems, as they provide us more certain values of attribute weights. We have suggested a new Sine entropy in IF settings in the current study. Also, a new MADM method is also suggested based on the suggested measure with help of numericals. Moreover, our suggested methods are more effective in allotting attributes weights and more promising results can be obtained.

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